Whistler Waves

Term Paper Presentation
Astrophysical Fluids and Plasmas Course

Vinay Kumar, 8 November 2022

Observations and Inferences

The first part of my talk will concern with the observations of whistlers and the inferences that can be drawn from these. In the second part of my talk I will briefly describe a theoretical model that explains these.

Lets first listen to whistler waves - or whistlers as they are often called. READ. And this is how the spectrum of these waves looks like. So lemme explain what's plotted. On the x-axis, we have time and on the y axis we have frequencies. The colours represent the amplitude of the wave, red being the loudest and blue being the most feeble of the lot.

Whistler Waves

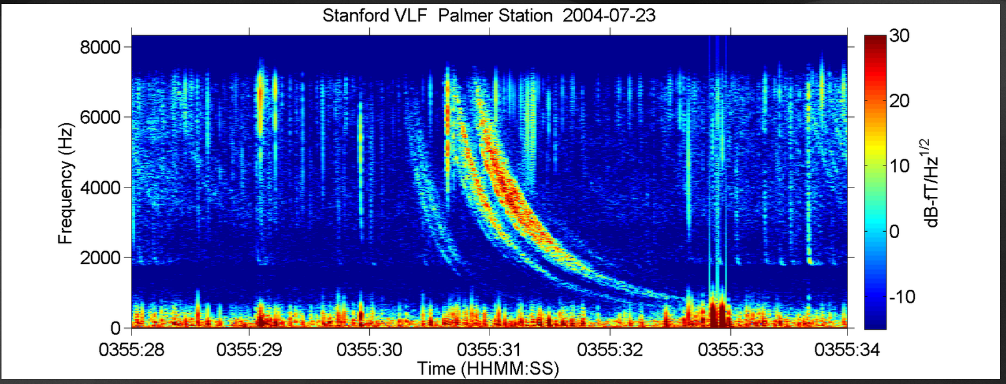
Whistler Waves

Whistler Waves

- Whistlers - radio waves in the audio frequency band.
- They sound like whistles with declining pitch.

Whistler Waves

- Whistlers - radio waves in the audio frequency band.
- They sound like whistles with declining pitch.



VLF spectrogram of an electromagnetic whistler wave, as received by the Stanford University VLF group's wave receiver at Palmer Station, Antarctica.

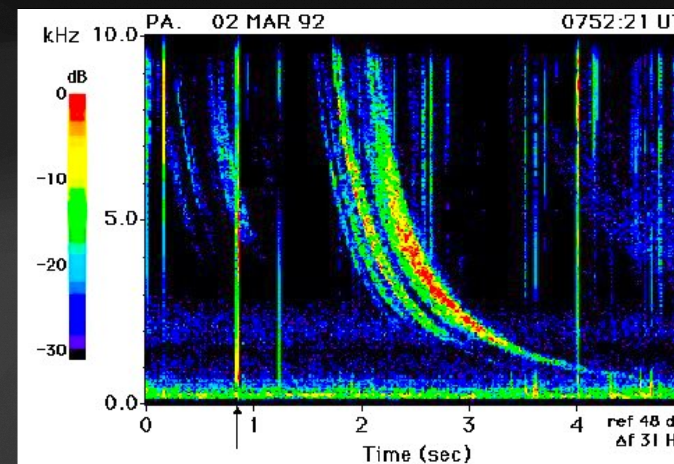
Terminology

- **Sferics** - Broadband electromagnetic pulses originating due to atmo*spheric* phenomena. These are heard as clicks and pops on a radio receiver.
- **Long Whistlers** - Whistlers heard after a sferic.
- **Short Whistlers** - Whistlers not preceded by a sferic.
- **Whistler Trains** - A group of whistlers heard one-after the other with uniform time gap between each.
- **Echoes** - All except the first member of a whistler train.

Sferics are Broadband electromagnetic pulses originating due to atmospheric phenomena. Broadband here means that their frequency range in very high - they have all modes excited - from low to high frequencies. They are heard as clicks or pops on a radio receiver. Read the rest.

Observations

Relation to Sferics



Spectrogram showing a whistler after a sferic. Source: Stanford VLF Group.

As I mentioned, there are these so called long whistlers which follow sferics - as can be seen in this plot. The small arrow near the x axis shows the instant the sferic was heard - and u can see that all frequencies have quite loud amplitudes at that time. Following this sferic, we have a whistler - a long whistler.

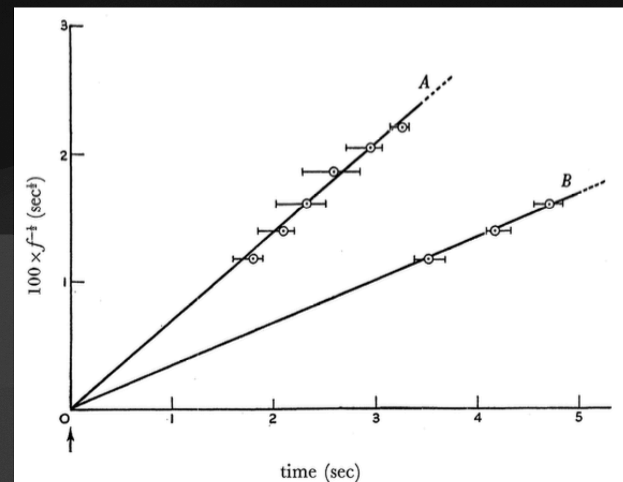
Properties of Whistlers

- Whistling tones with declining pitch - higher frequencies are heard before lower ones. High $\omega \implies$ High v.
- Long whistlers follow lightning strikes.

By now we know two things about whistlers. One, the declining tone of the whistle implies that high frequencies are heard before lower ones, which means that high frequency waves travel faster This is of course assuming that all frequencies were excited simultaneously at the same point in space aaand This might very well be the case for long whistlers, since they seem to follow lightning strikes.

Observations

Whistler Dispersion - Long Whistlers



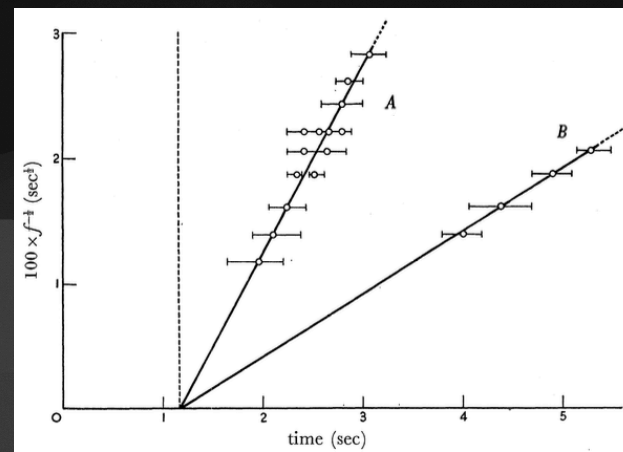
$\omega^{-1/2}$ vs t plot for two whistlers. A was heard before B
Source: Storey 1953

- Dispersion, D - the slope inverse of the graph b/w $1/\sqrt{\omega}$ and t.
- Echoes have a higher dispersion.
- Here, $D(B) = 2D(A)$.
- Intercept the t-axis at the time of the sferic - in case of long whistlers.
- In general, dispersions are in the ratio of 1:2:3:4.

If we plot the 1/root_frequency vs time graph for a long whistler, we see a straight line which intercepts the time axis at the time of the preceding sferic. The graph line for the echo also seems to trace back to the time of the sferic. We define the slope inverse of this graph as the dispersion of a whistler. What's observed is that the dispersion for the echoes occur in the ratio 1:2:3:4. And so on. All this is true for long whistlers.

Observations

Whistler Dispersion - Short Whistler



$f^{-1/2}$ vs t plot for two short whistlers. Source: Storey 1953

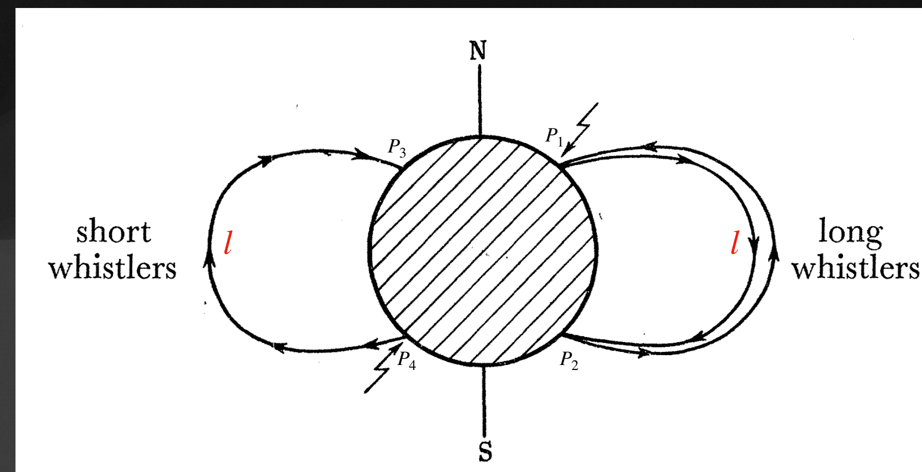
- In this case, $D(B) = 3D(A)$.
- Intercept the t-axis at the same time.
- Caused due to a sferic which was not heard?
- In general, dispersions are in the ratio 1:3:5:7.

For a short whistler, we don’t really have a sferic, but if the 1/root_omega vs t graphs are traced back for a whistle train, they intercept at a common point. Maybe there exists a sferic which was not heard for some reason? We’ll see. Also, for short whistler trains, the dispersions of the echoes go in the ratio 1:3:5:7.

Properties of Whistlers

- Whistling tones with declining pitch - higher frequencies are heard before lower ones. High $\omega \implies$ High v.
- Long whistlers probably do follow lightning strikes. Short whistlers?
- Speed, $v \sim \sqrt{\omega}$.
- Dispersions follow specific ratios - 1:2:3:4 or 1:3:5:7.
- Dispersions are measures of some distance.

An Explanation



Suggested path for the two types of whistlers. Source: Storey 1953

Let P1, P2, P3, P4 be four points on the surface of the earth as shown. P1 P2 here are conjugate points - which is to say they are points on the opposite ends of a particular field line. P3 and P4 are also conjugates. Let the length of this field line be ‘L’. Let’s assume that we have a plasma wave which propagates along the magnetic field. If some event, say lightning, excites a wave at P1, it traverses along the magnetic lines of force to the other hemisphere - where it gets reflected and comes back. So if you are sitting near P1, at first you’d receive a sferic from the lightning strike. Following this, after some time you’ll hear the waves which have traversed a distance of 2l. This would keep going on and on and you’ll hear the wave repeatedly as it traverses distances which are even multiples of ‘l’. Now assume that the motion of these waves is dispersive such that v goes as root omega. As a result we’ll get a spectrogram that matches with the observations. And the dispersion - since it depends on the distance the wave has travelled, would be in the ratio 2:4:6:8 which is like 1:2:3:4. This explains the long whistlers - if we can show that there actually exist waves which move along the magnetic field direction with a dispersion v ~ root omega.

This explanation can easily be extended for short whistlers. The claim here is that short whistlers are heard at conjugate points to where the lightning strikes. The distances traversed are odd multiples of l and hence explain the ratio of the dispersions. So all this is fine. Let us now try to see if such waves exist or not.

Theoretical Model

Model

- Plasma is cold $\implies T_{e,i} = 0$, and quasi-neutral.
- Stationary ions, electrons carry current $\implies \mathbf{j} = -en\mathbf{v}$.
- Quasi-Longitudinal approximation
  - Exclude displacement currents
  - Valid for $\omega < \omega_{ce} \ll \omega_{pe}$
- For Magnetosphere, $\omega_{ce} \sim 10^7$ Hz and $\omega_{pe} \sim 10^5$ Hz.
- Uniform Magnetic Field $\mathbf{B}_0\hat{z}$, transverse waves.

Model

Electron MHD Equations

$$\partial_t \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v} = \frac{q}{m_e} (\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j}$$

$$\nabla \times \mathbf{E} = -\partial_t \mathbf{B}$$

$$\nabla \cdot \mathbf{B} = 0$$

Linearize about

$\mathbf{B} = B_0 \hat{z}$; $\mathbf{v} = 0 + \mathbf{v}_1$

$$m_e \partial_t \mathbf{v}_1 = q \left(\mathbf{E}_1 + \mathbf{v}_1 \times B_0 \hat{z} \right)$$

$$\nabla \times \mathbf{B}_1 = \mu_0 n_0 q \mathbf{v}_1$$

$$\nabla \times \mathbf{E}_1 = -\partial_t \mathbf{B}_1$$

$$\nabla \cdot \mathbf{B} = 0$$

Model

Dispersion Relation

- We consider waves oscillating in the transverse direction to the uniform magnetic field, travelling along the magnetic field.
- Substitute $\mathbf{v}(z,t) = \mathbf{v}_0 e^{ikz - i\omega t}$ into the linearized equations.
- We get the dispersion relation:

$$k^2 - \frac{\omega_{ce}}{\omega} k^2 + \frac{\omega_{pe}^2}{c^2} = 0$$

- On using the approximations mentioned earlier, we can simplify this further to:

$$k = \frac{\omega_{pe}}{c} \sqrt{\frac{\omega}{\omega_{ce}}}$$

Model

Wave Velocity

- From the dispersion relation, we can get both the phase and the group velocities as

$$v_p = c\sqrt{\frac{\omega\omega_{ce}}{\omega_{pe}^2}}$$

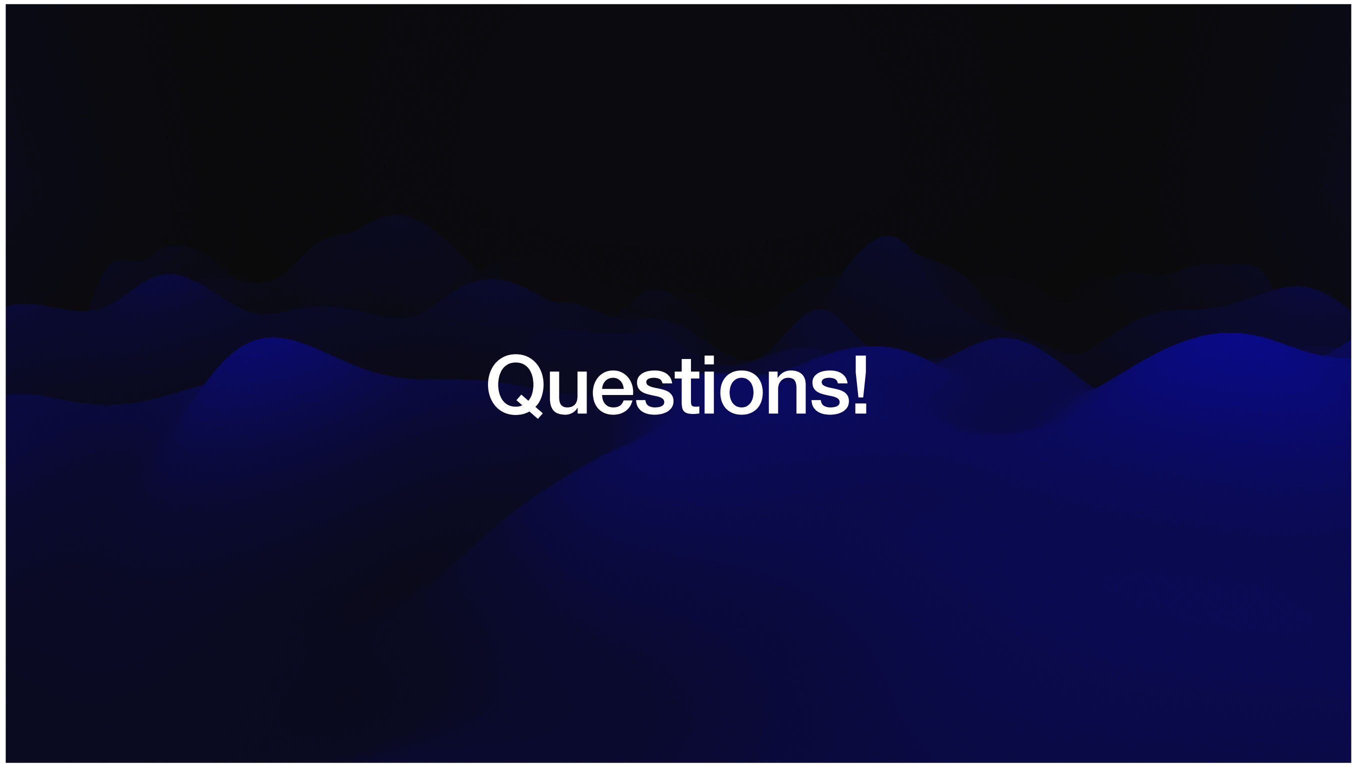
$$v_g = 2c\sqrt{\frac{\omega\omega_{ce}}{\omega_{pe}^2}}$$

- As can be seen, both of these $\sim \sqrt{\omega}$.

There are whistler waves!!

Summary

- “Heard” what whistlers are.
- Inferred properties of whistlers from observations.
- Put forward a possible explanation for their origin and detection.
- Saw how EMHD provides us with this explanantion.



That’s all from me today. Thank you.

International Centre for Theoretical Sciences

Astrophysical Fluids and Plasmas

PHY435.5

Term Paper: Whistler Waves

Vinay Kumar

30 November 2022

ICTS

1 Introduction

Whistlers, or whistler waves, as they are now called, were first heard by German ‘eavesdroppers’ during World War I while trying to intercept radio communications amongst the Allies. These waves have frequencies in the audible region of the VLF frequency band and sound like whistles of descending pitch - hence the name whistlers. The first part of this paper outlines the observations of whistler waves and and lists properties of whistlers which can be inferred from these observations. The second part of the paper proposes a theoretical model which predicts the existence and the properties of whistlers listed in the first part - thus explaining this mellifluous phenomenon.

2 Observations and Inferences

2.1 Associated Phenomena

Whistlers are usually either preceded by or followed by one or more of the following.

- **Sferics** (or "atmo*spherics*") : impulsive signals emitted by lightning. The frequency range is from a few hertz to millions of hertz. The audible part of this range has frequencies up to about 15 thousand hertz (15 kHz). On a spectrogram, sferics are characterized by vertical lines on the frequency-time graph indicating the simultaneous arrival of all of the audio frequencies. The sound of sferics consists of sharp crackling noises like twigs snapping or sizzling noises like bacon frying. Sferics are caused by lightning strokes within a couple of thousand kilometers of the receiver.

- **Whistler** : a musical tone, generated in the atmosphere, that descends in frequency over a few seconds, sounding just like a whistle that's fading away. On a spectrogram, whistlers appear as long sweeping arcs showing the sequential arrival of the frequencies.

- **Whistler Trains** - A group of whistlers heard one-after the other with uniform time gap between each.

- **Echoes** - All except the first member of a whistler train.

- **Choruses** - sound like many birds calling in turn. These are heard very occasionally, particularly during quiet times.

Shown in figure 1 is a spectrogram showing these phenomena.

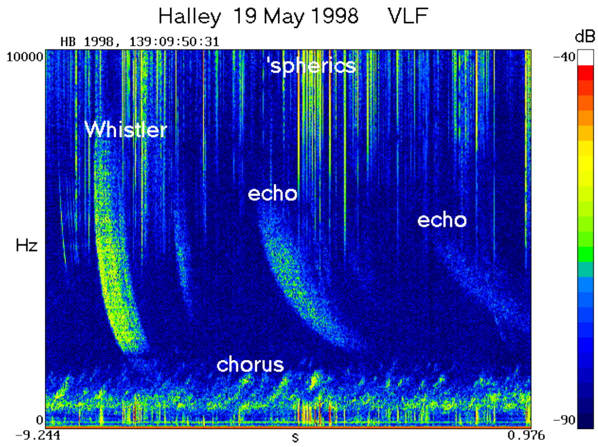


Figure 1: Spectrogram depicting sferics, whistlers, echoes and choruses. *Credit*: The British Antarctic Survey VLF Database.

2.2 Dispersion of a whistler

At this point one would like to quantify, in some way, the rate at which the pitch of the whistler descends. From spectrograms similar to figure 1, it was seen that the frequency, ω, vs time,t, spectrogram follows a trend like

$$\omega^{-1/2} \propto t \tag{1}$$

$$\implies D\omega^{-1/2} = t, \tag{2}$$

where D is a proportionality constant, added conventionally to the left hand side. D here is called the dispersion of the wave since it gives a measure of the time difference between the arrival of any two frequencies. So a plot of $\omega^{-1/2}$ vs t would have a slope of D^{-1}, as can be seen in figure 2.

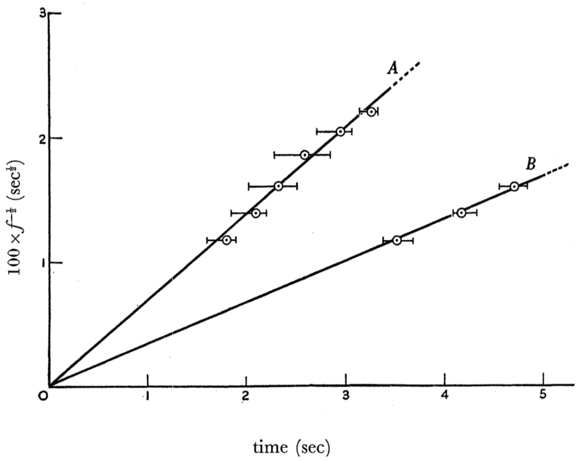


Figure 2: Plot of $\omega^{-1/2}$ vs t. Lines with gentler slope have larger dispersion. A is a whistler and B is the first echo.

Whistlers are found to fall into two distinct classes. A whistler either comes after a loud click - long whistler, or none at all - short whistler, never after a weak click. Atmospheric clicks do not produce whistlers on all occasions, but at a time of general whistler activity a loud click is always followed by a whistler. The fact that whistlers are often heard after a sferic, which has all frequencies excited, suggests a proposition, the validity of which will be examined bit by bit.

Proposition A: Whistlers are generated by sferics and travel some distance to reach a listener, and on the way get dispersed with a dispersion that is proportional to the distance travelled by them. The velocity of these waves is linearly dependent on $\sqrt{\omega}$ as can be inferred from plots like in figure 2.

This proposition is further supported by the observation that the $\omega^{-1/2}$ vs t plot for a set of whistler trains, no matter short or long, intercept the time axis at exactly the same point - which in fact coincides with the instant when the sferic was heard for long whistlers.

2.3 Whistler Trains

The dispersions for a train of long whistlers are found to be in a ratio of 1:2:3:4 and so on, whereas for short whistlers this ratio is 1:3:5:7. This suggests that successive echoes for long whistlers have travelled an integral multiple of some distance, say l_l, before being heard. On the other hand, echoes of short whistlers travel odd-integral multiples of some other length, l_s. With this information, we update our previous proposition as follows.

Proposition B: Whistlers are generated by sferics and travel some distance to reach a listener, and on the way get suitably dispersed. The velocity of these waves is linearly dependent on $\sqrt{\omega}$ as can be inferred from plots like in figure 2. Following a sferic, these waves travel along the magnetic field lines of the earth. They are then reflected back at the magnetically conjugate point to the source of the sferic and retrace their path to the source. Again, they are reflected back and go back to the conjugate point. This keeps happening till the energy dissipates and the disturbance is too feeble to be picked up by the radio receivers. A schematic for this is shown in figure 3.

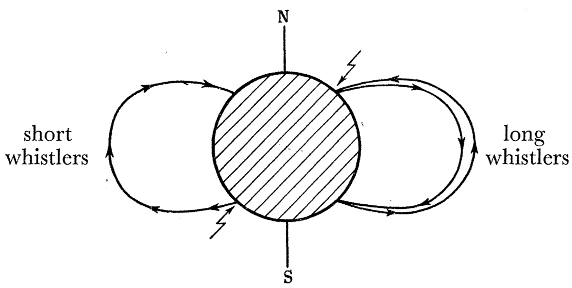


Figure 3: Suggested mechanism to explain whistlers.

This explains the ratio of dispersions of echoes for long and short whistlers. A listener near the source would first hear the sferic, following which they would hear a long whistler which has travelled a distance of l_l = 2OP, where OP is the distance between the source of the sferic, O, and the conjugate point, P.

Subsequently, the disturbance would bounce back again. As a result, the listener would get echoes which have travelled distances ~ 2OP, 4OP, 6OP and so on, thus matching with the observationally seen integral ratio. Short whistlers, on the other hand, are observed when the source and the receiver are at conjugate points. As a result, the first disturbance that the receiver gets is a dispersed wave which has travelled a distance of OP. Following this, the receiver gets pulses which have transversed distances ~ 3OP, 5OP, 7OP, again matching with the observationally seen odd-integral ratio for the dispersions of echoes.

So Proposition B is a pretty convincing one since it explain all the observationally seen properties of whistlers. In the next section, we shall see that there are indeed plasma waves which satisfy properties mentioned in Proposition B thus concluding our comprehensive explanation of this phenomenon.

3 Theory

3.1 The Electron MHD Model

The model used to describe the existence of whistler waves consists of the momentum equation for electrons with the standard Maxwell equations.

$$\frac{\mathrm{D}\mathbf{V}}{\mathrm{D}t} = -\frac{e}{m_e}\left(\mathbf{E} + \frac{\mathbf{V}\times\mathbf{B}}{c}\right), \tag{3}$$

$$\nabla\times\mathbf{B} = \frac{4\pi}{c}\mathbf{j} = -\frac{4\pi}{c}en\mathbf{V}, \tag{4}$$

$$\nabla\cdot\mathbf{B} = 0, \tag{5}$$

$$\nabla\times\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}. \tag{6}$$

Here, in equation 4, we have assumed that the current is carried only by electrons. The plasma is considered to be quasi-neutral, therefore the plasma density is constant in time and the model does not include the electron density continuity equation. The model is further simplified by the 'quasi-longitudinal' approximation of Electron Magnetohydrodynamics (EMHD). The quasi-longitudinal approximation simplifies the equations by excluding the displacement current in Ampere's Law (equation 6). The quasi-longitudinal approximation is valid for waves that satisfy the condition $\omega < \omega_{ce} \ll \omega_{pe}$ where ω is the angular frequency of the wave, ω_{ce} is the electron gyrofrequency, and ω_{pe} is the electron plasma frequency. The second inequality here holds for the Earth's magnetosphere.

We will further assume that the background magnetic field is uniform, $\mathbf{B} = B_0\hat{z}$ and consider stationary plasma, $\mathbf{V}_0 = 0$ as the equilibrium configuration. Further we will be concerned with transverse plasma waves which propagate along the magnetic field, ie. along the $\hat{z}$ direction, and result in variations perpendicular to the z-axis. With this, equations 3 - 6 can be linearized to get

$$\frac{\partial \bar{\mathbf{V}}}{\partial t} = -\frac{e}{m_e}\left(\bar{\mathbf{E}} + \frac{\bar{\mathbf{V}} \times B_0 \hat{z}}{c}\right), \tag{7}$$

$$\nabla \times \bar{\mathbf{B}} = -\frac{4\pi}{c} e n_0 \bar{\mathbf{V}}, \tag{8}$$

$$\nabla \cdot \bar{\mathbf{B}} = 0, \tag{9}$$

$$\nabla \times \bar{\mathbf{E}} = -\frac{1}{c}\frac{\partial \bar{\mathbf{B}}}{\partial t}. \tag{10}$$

Now, we take the curl of equation 7 twice, while substituting Ampere's Law and Faraday Law to get

$$\frac{\partial\left(\nabla \times \nabla \times \bar{\mathbf{V}}\right)}{\partial t} = -\frac{e}{m_e}\left(\frac{4\pi}{c^2}\frac{\partial n_0 e \bar{\mathbf{V}}}{\partial t} + \nabla \times \nabla \times \left(\frac{\bar{\mathbf{V}} \times B_0 \hat{z}}{c}\right)\right) \tag{11}$$

$$= -\frac{4\pi n_0 e^2}{c^2 m_e}\left(\frac{\partial \bar{\mathbf{V}}}{\partial t}\right) - \frac{e B_0}{m_e c}\left[\nabla \times \nabla \times \left(\bar{\mathbf{V}} \times \hat{z}\right)\right] \tag{12}$$

$$= -\frac{\omega_{pe}^2}{c^2}\left(\frac{\partial \bar{\mathbf{V}}}{\partial t}\right) - \omega_{ce}\left[\nabla \times \nabla \times \left(\bar{\mathbf{V}} \times \hat{z}\right)\right]. \tag{13}$$

Further, dividing both sides by ω_{ce}, normalizing time by setting $\omega_{ce} t = \tau$ and redefining $\frac{\omega_{pe}^2}{c^2}$ as some constant α, we get

$$\frac{\partial}{\partial \tau}\left(\alpha \bar{\mathbf{V}} + \nabla \times \nabla \times \bar{\mathbf{V}}\right) = -\nabla \times \nabla \times \left(\bar{\mathbf{V}} \times \hat{z}\right). \tag{14}$$

3.2 Obtaining the Dispersion Relation, Phase and Group Velocities

We introduce perturbations of the form $\vartheta = \vartheta_0 e^{i(kz+\omega t)} = \vartheta_0 e^{i\left(kz + \frac{\omega}{\omega_{ce}}\tau\right)}$ since we wish to look at transverse waves traveling along the z-axis. Subbing this in the equation above, we get

$$i\frac{\omega}{\omega_{ce}}\left(\alpha + k^2\right)\bar{\mathbf{V}} = -k^2\left(\bar{\mathbf{V}} \times \hat{z}\right). \tag{15}$$

This equation relates $\bar{\mathbf{V}}$ to $\bar{\mathbf{V}} \times \hat{z}$. We wish to eliminate $\bar{\mathbf{V}}$ so as to get the dispersion relation. For this we cross the above equation with $\hat{z}$. This lends us

$$i\frac{\omega}{\omega_{ce}}\left(\alpha + k^2\right)\alpha \hat{z} \times \bar{\mathbf{V}} = -k^2 \bar{\mathbf{V}}. \tag{16}$$

Writing the preceding two equations in a matrix form, we get

$$\begin{pmatrix} i\frac{\omega}{\omega_{ce}}\left(\alpha + k^2\right) & -k^2 \\ -k^2 & -i\frac{\omega}{\omega_{ce}}\left(\alpha + k^2\right) \end{pmatrix}\begin{pmatrix} \bar{\mathbf{V}} \\ \bar{\mathbf{V}} \times \hat{z} \end{pmatrix} = 0. \tag{17}$$

Now setting the determinant of the matrix to zero, we get the dispersion relation as

$$\left(\frac{\omega}{\omega_{ce}}\left(\alpha+k^2\right)\right)^2-k^4=0 \tag{18}$$

$$\implies \frac{\omega_{ce}}{\omega}k^2=\alpha+k^2 \tag{19}$$

Using the quasi longitudinal approximation, we have

$$k=\frac{\omega_{pe}}{c}\sqrt{\frac{\omega}{\omega_{ce}}}. \tag{20}$$

From this, we get the phase and the group velocities as

$$v_p=\frac{\omega}{k}=\sqrt{\frac{\omega\omega_{ce}}{\omega_{pe}^2}}, \tag{21}$$

$$v_g=\frac{\mathrm{d}\omega}{\mathrm{d}k}=2\sqrt{\frac{\omega\omega_{ce}}{\omega_{pe}^2}}. \tag{22}$$

Both the phase and the group velocities grow linearly with $\sqrt{\omega}$. So the EMHD theory does indeed predict plasma waves that travel along magnetic field lines which travel with velocities which $\sim\sqrt{\omega}$. And so, we conclude that it is these plasma waves that are heard as whistlers.

4 Discussion and Concluding Remarks

In this paper, we described the existence and properties of whistlers from an observational viewpoint and finally outlined a theory to prove their existence. Whistlers are an important probe to study the magnetic fields around various astrophysical objects. Many present studies are underway inferring the magnetic field configurations of Jupiter and Saturn which use whistler data from satellites in the respective atmospheres to look for whistlers and infer about the magnetic fields there.

References

[1] "An investigation of whistling atmospherics" (1953) Philosophical Transactions of the Royal Society of London. Series A, Mathematical and Physical Sciences, 246(908), pp. 113–141. Available at: https://doi.org/10.1098/rsta.1953.0011.