Exploring the Tearing Mode Instability in 3D using Direct Numerical MHD Simulations

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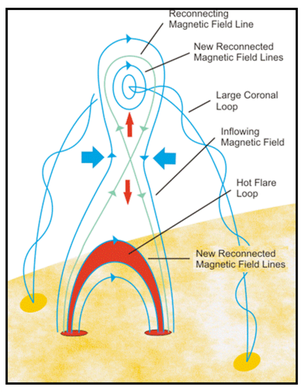
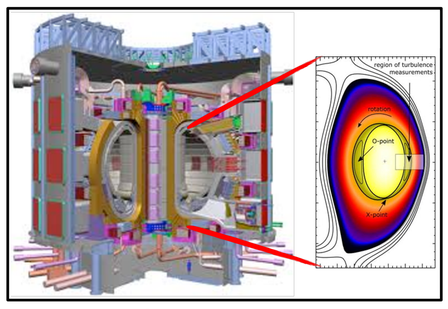
Magnetic Reconnection (MR)

Solar Flares



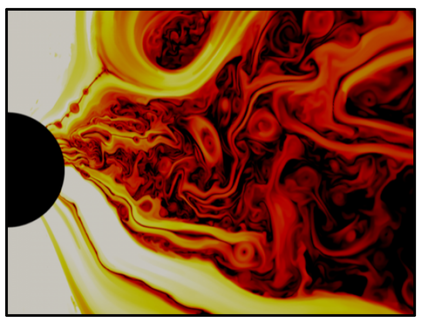
Credits: NASA

Fusion Reactors



Credits: ITER (left), L. Bardóczi et al (right)

Accretion Disks

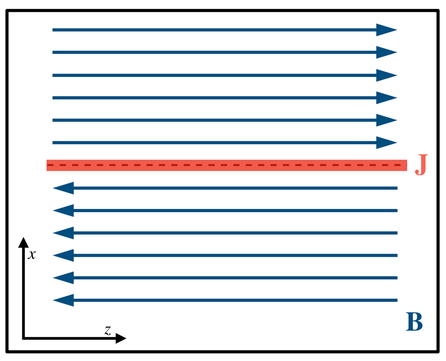


Credits: Ripperda et al

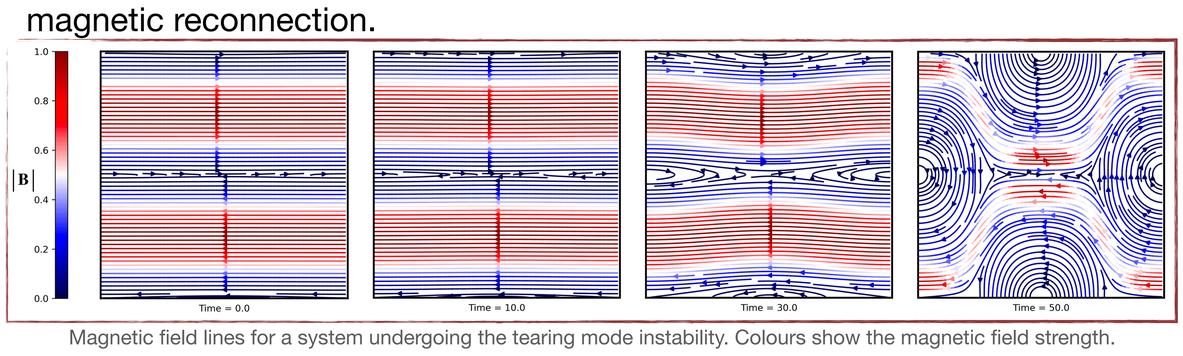
- MR involves a violent topological rearrangement of antiparallel magnetic fields leading to the redistribution of magnetic energy to kinetic energy, heating, and particle acceleration.
- This phenomenon cannot occur in ideal MHD as it breaks flux freezing. This is the case when $\mathbf{E} + \mathbf{v} \times \mathbf{B} \neq 0$.

Tearing Mode Instability (TMI)

- One of the simplest non-ideal MHD instability is the TMI, where resistivity, η, is the non-ideal ingredient, $\mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta \mathbf{J}$.
- It has been studied extensively in 2.5D, starting from the seminal work of Furth et al [1] in 1963, and Coppi et al [2] in 1976.

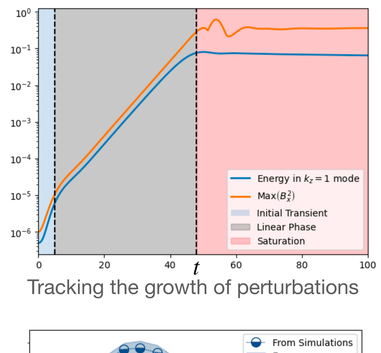


- The equilibrium configuration consists of reversing magnetic field lines across an interface, giving rise to a current sheet, say, of a thickness, a, which is unstable to perturbations along z, which grow, and subsequently undergo magnetic reconnection.

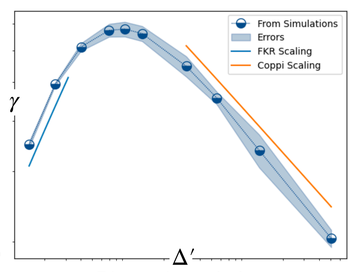


Magnetic field lines for a system undergoing the tearing mode instability. Colours show the magnetic field strength.

- The instability of a mode with wave number k is controlled by the instability parameter, Δ', which depends on the aspect ratio, ak, and the form of the equilibrium.
- Modes for which $\Delta' > 0$ are unstable.
- The growth rate, γ, is also controlled by Δ' along with the Lundquist number, S, and has two regimes:
- FKR regime: wherein γ increases with increasing Δ'.
- Coppi regime: wherein γ decreases with increasing Δ'.



Tracking the growth of perturbations



Dispersion relation

What happens in 3D?

- The effect of dimensionality on TMI has been relatively less explored.
- We initiate a tearing-like instability in 3D and estimate the growth rates, and the dispersion relation in order to systematically compare 2.5D and 3D TMI.
- For this, we choose the $\sin$ equilibrium in 2.5D, $\mathbf{B}_0(x, z) = \sin(x)\hat{\mathbf{z}}$
- The extension to 3D can be done in two ways:
  - 3DA: Uniformly extending the 2.5D equilibrium to 3D, $\mathbf{B}_0(x, y, z) = \sin(x)\hat{\mathbf{z}}$,
  - 3DB: Including a modulation along the perpendicular direction, $\mathbf{B}_0(x, y, z) = \sin(x)/\cosh^2(y)\hat{\mathbf{z}}$.

Equations + Setup

Incompressible Resistive MHD

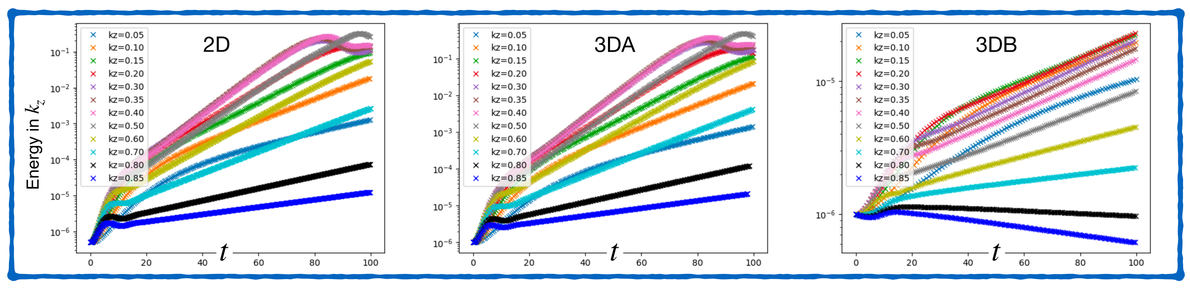
Induction Equation: $\dfrac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$

Momentum Equation: $\dfrac{D\mathbf{V}}{Dt} = -\nabla p + \mathbf{J} \times \mathbf{B} + \nu \nabla^2 \mathbf{V}$

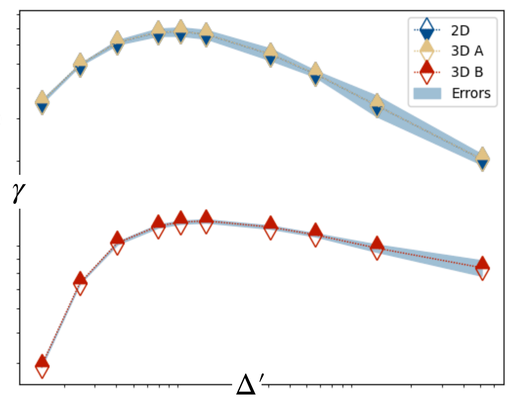
Incompressibility: $\nabla \cdot \mathbf{V} = 0$ Gauss' Law: $\nabla \cdot \mathbf{B} = 0$

- Simulations are performed using a pseudospectral code - Dedalus [3].
- The equilibrium is perturbed by a $\sin$ wave along z with a wavenumber k_z.
- We solve the above equations in a triply periodic box of dimensions $[-\pi, \pi] \times [-\pi, \pi] \times [-\pi/k_z, \pi/k_z]$ with a resolution of 192 Fourier modes along each direction.
- The Lundquist number ($S \sim 1/\eta$) is taken to be 100, and the Prandtl number ($Pm \sim \nu/\eta$) is taken to be 1.
- To prevent the diffusion of the equilibrium, the diffusion term for the equilibrium is removed from the induction equation at all times, as explained in Landi et al [4].

Simulation Results - Growth Rates



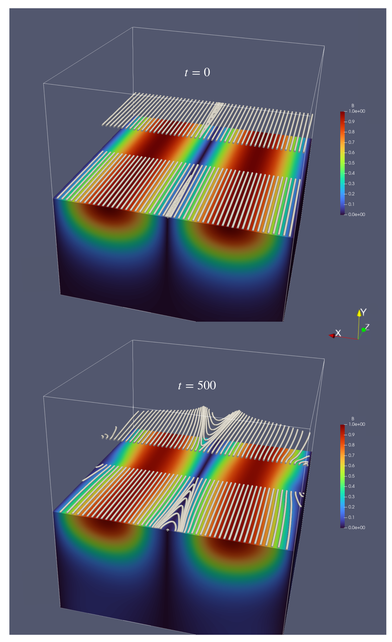
- For the uniformly extended current sheet (case 3DA), we find that there is no change in the growth rate of the TMI as we go from 2D to 3D.
- This can be seen as a simple validation of the Squire's theorem [5].



- For the modulated case (3DB), there is a clear reduction in the growth rate as compared to the 2D case. Some 2D unstable modes even become stable.
- The fastest growing mode, however, stays the same for all the three cases.

Simulation Results - Island Widths

- The plot alongside shows the evolution of magnetic structures. Colours represent field strength. White lines are the magnetic field lines.
- As can be seen, the magnetic field lines indeed have a full three dimensional structure - they are not constrained to any one plane - pointing to a clear distinction between the 2D and the 3D cases.
- The island width also varies along the y direction - it is minimum in the mid plane, and increases as we go toward the boundaries.



References

[1] H. P. Furth, J. Killeen, and M. N. Rosenbluth, *Phys. Fluids* 6, 459–484 (1963).
[2] B. Coppi et al., *Sov. J. Plasma Phys.* 2, 533–535 (1976).
[3] K. J. Burns et al., *Phys. Rev. Res.* 2, 023038 (2020).
[4] S. Landi et al., *Phys. Plasmas* 15, 012302 (2008).
[5] H. B. Squire, *Proc. R. Soc. Lond. A* 142, 621-628 (1933).

Scan for audio description.

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